

Testing documentation

Requirements and Analyzes

Double integrator

Version: 1.2

Goal:

The goal of this document is to describe function „Double integrator“ which consists of the function „Integrator with state boundaries“ and to explain the reasoning behind the test design which was developed. Testing documentation for the „Integrator with state boundaries can be found here [..\integrator\_with\_state\_boundaries\Testing documentation.docx](../integrator_with_state_boundaries/Testing%20documentation.docx) If the function is developed according to the specifications the test will have a value of True.

System description:

Double integrator is a function that has seven inputs, namely integration input value, limits for the first integrator, limits for the second integrator and initial values for both integrators. The function has two outputs, one for each integrator, where the output value of the first integrator becomes input value for the second integrator.

In our example, acceleration is the value that is integrated in the first integrator and it is limited with maximum and minimum acceleration. Next three inputs are used to calculate valid value of the velocity, which is then used as an input to the second integrator, used to calculate displacement.

The integral of the acceleration function a(t) is the velocity function v(t); that is, the area under the curve of acceleration vs. time (a vs. t) graph corresponds to velocity.

The integral of the velocity function v(t) is the displacement function r(t); that is, the area under the curve of velocity vs. time (v vs. t) graph corresponds to displacement.

Both integrators use the same integration method, namely Euler Forward Integration. With the first time step, block state n = 0, with either initial output y(0) = IC or initial state x(0) = IC, depending on the Initial condition setting parameter value.

For a given step n > 0 with simulation time t(n), Simulink® updates output y(n) as follows:

Source : <https://www.mathworks.com/help/simulink/slref/discretetimeintegrator.html>

Sample time is constant throughout the simulation and represented with Ts, so the formula above can be representend as:

Figure 1 shows the connection between input acceleration, limited acceleration, and both outputs, in respect to initial values of velocity and displacement as well as limits for both acceleration and velocity.

A picture containing object

Description generated with very high confidence

Figure 1: connection between function inputs and outputs

Test design:

The test is designed to check if the function is working exactly how it is described above. In order to validate function three requirements have to be satisfied (requirements are listed below).

The test uses differentiation as the reverse of integration.

Acceleration is the first derivative of velocity with respect to time:

Velocity is the first derivative of displacement with respect to time:

Acceleration is the second derivative of the displacement with respect to time, or the first derivative of velocity with respect to time:

Since measurnments are taken every Ts seconds, acceleration can be expressed as:

And velocity as:

Figure 2 shows calcultaed velocity and acceleration if the displacement function is an input. By comparing figure 1 and figure 2 it is visible that the velocity graph looks the same but the calculated acceleration graph differs both from the input value and the limited value of acceleration. The reason for that are velocity and acceleration limits.

A picture containing kite, flying

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Figure 2: connection between test inputs and outputs

Test requirements:

R1:

Velocity derivative over time has to be less or equal than maximum acceleration and greater or equal than minimum acceleration.

Where v(a\_min) represents the velocity that body has at the moment t(n+1) if it was moving with the minimal acceleration from the moment t(n). Simirarly, v(a\_max) represents the velocity that body has at the moment t(n+1) if it was moving with the maximum acceleration from the moment t(n).

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R2:

Displacement derivative over time has to be equal to the calculted velocity.

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Please note:

Glob truncation error in the Euler Forward integration is proportional to the time step (Ts). Since there is an error in numerical calculations, the values have to be almost equal, or in mathematical words:

where Ɛ→0

R3:

Displacement derivative over time has to be less or equal to the maximal velocity and greater or equal then minimal velocity.

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Description generated with high confidence

In order for the test to have the value of true, all of the requirements have to be true

T = R1 and R2 and R3

Test cases:

Case 1:

The body accelerates with the maximum acceleration until the moment when the velocity becomes maximum allowed. The body then starts deaccelerating with the minimum acceleration allowed until it reaches minimal velocity allowed.

Case 2:

The body deaccelerates with minimal acceleration until it reaches the minimal velocity after which it accelerates with maximum acceleration until it reaches maximum velocity.

Case 3: General case: